Plural Quantifications and Generalized Quantifiers

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Abstract. This paper discusses two important results on the expressive limitations of elementary languages that David Kaplan established a few decades ago, and clarifies how they relate to the expressive power of plural constructions of natural languages. Kaplan proved that such plural quantifications as the following cannot be paraphrased into elementary languages:

Most things are funny. (1)
Some critics admire only one another. (2)

The proof that (1) cannot be paraphrased into elementary languages is often taken to support the generalized quantifier approach to natural languages, and the proof that (2) cannot be so paraphrased is usually taken to mean that (2) is a second-order sentence. The paper presents an alternative interpretation: Kaplan's results provide important steps toward clarifying the expressive power of plural constructions of natural languages vis-à-vis their singular cousins. In doing so, the paper compares (regimented) plural languages with generalized quantifier languages, and plural logic with second-order logic.

Keywords: semantics, natural language, plural construction, plural logic, Geach-Kaplan sentence, generalized quantifier, Rescher quantifier, plural quantifier

1 Introduction

Plural constructions (in short, *plurals*) are as prevalent in natural languages as singular constructions (in short, *singulars*). This contrasts natural languages with the usual symbolic languages, e.g., elementary languages or their higher-order extensions.[1] These are singular languages, languages with no counterparts of natural language plurals, because they result from regimenting singular fragments of natural languages (e.g., Greek, German, or English).[2] But it is commonly thought that the lack of plurals in the usual symbolic languages result in no deficiency in their expressive power. There is no need to add counterparts of natural language plurals to symbolic languages, one might hold, because plurals are more or less devices for abbreviating their singular cousins: 'Ali and Baba are funny' and 'All boys are funny', for example, are more or less abbreviations of 'Ali is funny and Baba is funny' and 'Every boy is funny', respectively. But there are more recalcitrant plurals, plurals that cannot be considered abbreviations of singulars, e.g., 'The scientists who discovered the structure of DNA cooperated' or 'All the boys cooperated to lift a piano.' So I reject the traditional view of plurals as abbreviation devices, and propose an alternative view that departs radically from the tradition that one can trace back to Aristotle through Gottlob Frege. Plurals, on my view, are not redundant devices, but fundamental linguistic devices that enrich our expressive power, and help to extend the limits of our thoughts. They belong to *basic linguistic*

[1] What I call *elementary languages* are often called *first-order languages*. I avoid this terminology, because it suggests contrasts only with higher-order languages.

[2] Some natural languages (e.g., Chinese, Japanese, or Korean) have neither singulars nor plurals, because they draw no syntactic distinction between the two kinds of constructions. This does not mean that those languages, like the usual symbolic languages, have no counterparts of plurals or that they have no expressions for talking about many things (as such).

categories that complement the categories to which their singular cousins belong, and have a *distinct semantic function*: plurals are by and large devices for talking about *many* things (as such), whereas singulars are more or less devices for talking about *one* thing ('at a time').[3] It has been a few decades since David Kaplan established two important limitations of elementary languages by considering natural language plurals. In this paper, I ruminate on his results to clarify the expressive power of plurals.

2 Expressive Limitations of Elementary Languages

Elementary languages can be taken to have five kinds of primitive expressions:

(a) *Singular Constants*: 'a', 'b', etc.
(b) *Singular Variables*: 'x', 'y', 'z', etc.
(c) *Predicates*
  (i) 1-place predicates: 'B^1', 'C^1', 'F^1', etc.
  (ii) 2-place predicates: '$=$', 'A^2', etc.
  (iii) 3-place predicates: 'G^3', etc.
  Etc.
(d) *Boolean Sentential Connectives*: '$-$', '$\wedge$', etc.
(e) *Elementary Quantifiers*: the singular existential '$\exists$', and the singular universal '$\forall$'

The constants amount to proper names of natural languages, e.g., 'Ali' or 'Baba'; the variables to singular pronouns, e.g., 'he', 'she', or 'it', as used anaphorically (as in, e.g., 'A boy loves a girl, and *she* is happy'); the predicates to verbs or verb phrases in the singular form, e.g., 'is a boy', 'is identical with', 'admires', or 'gives ... to —'; and the quantifiers to 'something' and 'everything'. 'Something is a funny boy', for example, can be paraphrased by the elementary language sentence '$\exists x[B(x) \wedge F(x)]$', where '$B$' and '$F$' are counterparts of 'is a boy' and 'is funny', respectively.[4] Elementary languages, note, have no quantifier that directly amounts to the determiner 'every' in, e.g., 'Every boy is funny', but this determiner can be defined in elementary languages, as is well-known. 'Every boy is funny', for example, can be paraphrased by the universal conditional '$\forall x[B(x) \rightarrow F(x)]$', which amounts to 'Everything is such that if it is a boy, then it is funny.'

Now, say that natural language sentences (e.g., 'Something is a funny boy', 'Every boy is funny') are *elementary*, if they can be paraphrased into elementary languages. Kaplan showed that the following plural constructions are *not* elementary:

Most are funny. (1a)
Most boys are funny. (1b)
Some critics admire only one another.[5] (2)

He showed that these cannot be paraphrased into elementary languages with predicates amounting to 'is a boy' and 'is funny' (e.g., 'B' and 'F'). An important feature of elementary languages that one can use to obtain such a result is that their logic, elementary logic, is *compact*, that is,

[3] See, e.g., Yi [17]–[20] for an account of plurals based on the view sketched in this paragraph.

[4] The superscript of a predicate is omitted, if the number of its argument places is clear from the context.

[5] (2) can be paraphrased by 'There are some critics each one of whom is a critic, and admires something only if it is one of them but is not identical with him- or herself.'

Compactness of elementary logic:

> If some sentences of an elementary language $\mathscr{L}$ logically imply a sentence of $\mathscr{L}$, then there are finitely many sentences among the former that logically imply the latter.

So if a sentence is logically implied by infinitely many elementary sentences, but not by any finitely many sentences among them, this means that the sentence is not elementary.

Now, consider (1a) and (1b). Rescher [12] considers two quantifiers that roughly amount to the two uses of 'most' in these sentences. (1a) might be considered an abbreviation of a sentence in which a noun (in the plural form) follows 'most' (e.g., (1b) or 'Most things are funny'). But one might regard 'most' in (1a) as a *unary* quantifier, one that, like 'everything', can combine with one (1-place) predicate (in the plural form) to form a sentence. Rescher proposes to add to elementary languages a new quantifier, '*M*', that corresponds to 'most' in (1a), so construed. Using the quantifier, which came to be known as (Rescher's) *plurality quantifier*,[6] he puts (1a) as follows:

$$MxF(x) \tag{1a*}$$

where '*F*' is an elementary language predicate amounting to 'is funny'. He takes the quantification $Mx\phi(x)$ to say that "the set of individuals for which ϕ is true has a greater cardinality than the set for which it is false" ([12], 373). And he states that the quantifier '*M*' cannot be defined in elementary languages (*ibid*., 374), which means that (1a) and the like are not elementary,[7] but without indication of how to prove it. The statement was proved by Kaplan [6].[8] Here is a simple proof: '$-MxF(x) \wedge Mx[F(x) \vee x{=}y]$' is satisfiable in any finite model, but in no infinite model; but elementary languages, whose logic is compact, have no such sentence.[9]

In response to the Rescher-Kaplan result, some might argue that the quantifier 'most' (or '*M*') is not definable in elementary languages because it is not a logical but a mathematical expression. They might hold that (1a), for example, is a statement about numbers or sets: it means that the *number* of funny things is greater than the *number* of non-funny things (or that the *cardinality* of the *set* of funny things is greater than the *cardinality* of the *set* of non-funny things). If so, they might conclude, the Rescher-Kaplan result merely confirms the well-known limitations of elementary languages in expressing mathematical truths, e.g., the mathematical induction principle, which helps to give a categorical characterization of arithmetical truths.

Although it might seem plausible that 'most' has an implicit reference to numbers or sets, it does not seem plausible to hold the same about 'some' in (2). This sentence does not seem to pertain to mathematics at all. So Boolos says that (2) and the like, unlike (1), '*look* as if they 'ought to' be symbolizable' in elementary languages ([3], 433;

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[6] Rescher says that sentences with the quantifier '*M*' involve "the new mode of *plurality-quantification*", but calls the quantifier itself "M-quantifier" ([13], 373). Kaplan [6]–[7] calls it "the plurality quantifier".

[7] To relate the undefinability of '*M*' to the natural language quantifier 'most', it is necessary to assume that the former (as Rescher explains it) captures the latter. This is a controversial assumption; I think that 'most' is usually used to mean *nearly all*, rather than *more than half* or *a majority* (*of*). (Westerståhl [15] holds that it is an ambiguous expression that has two readings, which I doubt.) But we can take Rescher's plurality quantifier to correspond to 'more than half' or 'a majority', and the Rescher-Kaplan result to pertain to *this* quantifier.

[8] Kaplan [6] gives sketches of proofs of this and other interesting facts about languages that contain '*M*'. It is straightforward to define 'most' in (1a) in terms of the binary determiner 'most' in (1b); (1a) is equivalent to 'Most things that are identical with themselves are funny.' So Kaplan's proof of the undefinability of the former extends to the latter.

[9] Rescher ([13], 374) does not introduce a symbolic counterpart of the binary determiner 'most' in (1b), but he states that it cannot be defined in elementary languages or even their extensions that result from adding '*M*'. Barwise & Cooper ([2], 214f) prove this statement, and note that Kaplan proved the statement in 1965, but did not publish the proof. (Barwise & Cooper's proof yields a stronger result.)

original italics). But Kaplan proved that they are not.[10] (2), which came to be known as the *Geach-Kaplan sentence*, cannot be paraphrased into elementary languages with counterparts of 'is a critic' and 'admires' (e.g., 'C^1' and 'A^2').

His proof begins by paraphrasing (2) by a second-order sentence:

$$\exists^2 X\{\exists x X(x) \wedge \forall x[X(x) \rightarrow C(x)] \wedge \forall x \forall y[X(x) \wedge A(x, y) \rightarrow x \neq y \wedge X(y)]\} \tag{2a}$$

where 'X' is a second-order variable, and '$\exists^2$' the second-order existential quantifier. By replacing '$C(x)$' and '$A(x, y)$' in (2a) with '$N(x) \wedge x \neq 0$' and '$S(x, y)$', where 'N' and 'S' amount to 'is a natural number' and 'is a successor of',[11] respectively, we can get the following:[12]

$$\exists^2 X\{\exists x X(x) \wedge \forall x[X(x) \rightarrow N(x) \wedge x \neq 0] \wedge \forall x \forall y[X(x) \wedge S(x, y) \rightarrow x \neq y \wedge X(y)]\} \tag{2a*}$$

which amounts to 'Some non-zero natural numbers are successors only of one another.' So if (2a) can be paraphrased into elementary languages with 'C' and 'A', so can (2a*) into those with 'N' and 'S'. But (2a*) cannot be paraphrased into elementary languages; its negation is equivalent to the second-order mathematical induction principle, which cannot be expressed in them.[13]

Most of those who discuss Kaplan's proof take it to show that (2) also turns out to be a covert statement of a mathematical fact. It is commonly held that (2) *is* a second-order sentence, comparable to (2a), which has the second-order existential quantifier '$\exists^2$', and it is usual to take second-order quantifiers to range over sets (or classes), e.g., sets of critics.

But Kaplan's proof does not support the conclusion that (2) is a second-order sentence or a sentence that implies the existence of a non-empty set (or class) of critics. To see this, consider the following sentences:

> Ezra is a critic, Thomas is a critic, Ezra is not Thomas, Ezra admires only Thomas, and Thomas admires only Ezra. (2.1)
>
> Ezra and Thomas are critics who admire only one another. (2.2)

We can intuitively see that (2.1) logically implies (2.2), and that (2.1) logically implies (2). So (2.1) logically implies (2). Then (2) cannot imply the existence of a set because (2.1), which has straightforward paraphrases in elementary languages, does not do so.[14] And we can prove that (2) is not elementary without assuming that it can be paraphrased by a second-order sentence (e.g., (2a)). Consider the following series of infinitely many elementary language sentences:

> (3.1) c_1 is a critic who admires only c_2; (3.2) c_2 is a critic who admires only c_3; . . . .; (3.n) c_n is a critic who admires only c_{n+1}; . . . . (3)

where 'c_1', 'c_2', 'c_3', etc. are different proper names. We can intuitively see that these sentences, taken together, logically imply (2): if they hold, then c_1, c_2, c_3, etc. are critics who admire only one another. But no finitely many sentences among

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[10] Kaplan communicated his proof to Quine. Quine ([11], 238f) states Kaplan's result, and argues that (2) is a sentence about sets or classes of critics. Kaplan's proof is reproduced in Boolos ([3], 432f). See also Almog [1].

[11] A natural number x is said to *be a successor of* a natural number y, if $x=y+1$.

[12] (2a*) is slightly different from the arithmetical sentence used in Kaplan's proof. See the sentence (C) in Boolos (1984, 432). But it is straightforward to see that they are logically equivalent.

[13] The mathematical induction principle, added to the other, Dedekind-Peano axioms, which are elementary, yields a categorical characterization of arithmetical truths. But one cannot give a categorial characterization of arithmetical truths in an elementary language, which can be proved using the compactness of elementary logic.

[14] For an elaboration of this argument, see Yi ([18], Ch. 1). See also Yi [17] & [19].

them logically imply (2): (3.1)-(3.n), for example, do not logically imply that c_{n+1} is a critic who admires nothing but c_1, c_2, . . ., c_{n+1}. So (2) cannot be paraphrased into elementary languages, whose logic is compact.[15]

Now, some might think that (2) is non-elementary only because it concerns cases involving infinitely many things. Then those who think, plausibly or not, that we are not concerned with such cases outside mathematics might argue that elementary languages are powerful enough as long as we do not engage in higher mathematical enterprise, and restrict our domain of discourse to finite domains. It would be wrong to do so. There is no elementary language sentence that agrees with (2) *even on all finite domains*. We can show this by applying basic results of Finite Model Theory.[16]

3 From Singular Languages to Plural Languages

The Geach-Kaplan sentence (2), we have seen, cannot be paraphrased into elementary languages. This is usually taken to show that it is a second-order sentence that has an implicit quantification over sets of critics. But there is no good reason to take it to be a second-order sentence by taking the plural quantifier 'some' in the sentence as a second-order quantifier. Although Kaplan's proof of its non-elementary character proceeds by paraphrasing it by its second-order analogue, as we have seen, there are alternative, more direct proofs that do not rest on the paraphrase. By contrast, there is a clear contrast between the plural constructions involved in (2) and the singular constructions involved in, e.g., (2.1)–(2.2) and (3.1)–(3.n), which have straightforward elementary language counterparts. So there is a good reason to take Kaplan's result on (2) to show the limitations of singulars (especially those incorporated into elementary languages) vis-à-vis plurals.[17] If so, it would be useful to develop *plural extensions* of elementary languages, symbolic languages that have refinements of natural language plurals while containing elementary languages as their singular fragments, to give a theory of the *logical relations pertaining to plurals*, e.g., those that relate (2) to (2.1)-(2.2) or those that relate (2) to the sentences in (3). I have presented such languages by regimenting basic plural constructions of natural languages, and characterized their logic in some other publications.[18] Let me explain the basics of those symbolic languages, called (*first-order*) *plural languages,*[19] to show that they have natural paraphrases of plural constructions.

Plural languages extend elementary languages by including plural cousins of singular variables, predicates, and quantifiers of elementary languages:

(b*) *plural variables*: '*xs*', '*ys*', '*zs*', etc.[20]
(c*) *plural predicates*: '$\boldsymbol{C}^1$' (for '*to* cooperate'), 'H^2' (for 'is one of"), '$\boldsymbol{D}^2$' (for '*to* discover'), '$\boldsymbol{L}^2$' (for '*to* lift'), '$\boldsymbol{W}^2$' (for '*to* write'), etc.
(e*) *plural quantifiers*: the existential '$\boldsymbol{\Sigma}$', and the universal '$\boldsymbol{\Pi}$'

Plural variables are refinements of the plural pronoun 'they', as used anaphorically as in 'Some scientists worked in Britain, and *they* discovered the structure of DNA', where 'they' takes 'some scientists' as the antecedent. Plural

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[15] See Yi [21], which argues that logic is not axiomatizable, for an elaboration of this argument. See also Yi ([20], 262).

[16] Similarly, we can show that no elementary language sentence agrees with '$MxF(x)$' on all finite models. See Appendix.

[17] Kaplan's results do not suffice to show that we cannot accommodate natural language plurals into the usual, singular higher-order languages. But we can add to Kaplan's results other results that show this. See, e.g., Yi ([17], 172-4) and ([19], 472-6).

[18] See, e.g., Yi [17]–[20].

[19] The logic of plural languages is called *plural logic*.

[20] I add '*s*' to a lower-case letter of English alphabet to write a plural variable, but plural variables, like singular variables, are simple expressions with no components of semantic significance.

quantifiers, which bind plural variables, are refinements of 'some things' and 'any things'. And plural predicates are refinements of usual natural language predicates (e.g., '*to* discover'), which can combine with plural terms (e.g., 'they') as in the above-mentioned sentence, and have one or more argument places that admit a plural term, e.g., the only argument place of '$\boldsymbol{C}^1$', the first argument place of '$\boldsymbol{D}^2$', or the second argument place of '$\mathbf{H}^2$'.[21] Elementary language predicates, by contrast, are refinements of the *singular forms* of natural language predicates (e.g., 'is funny' or 'admires'), and have no argument place that admits plural terms; so they can combine only with singular terms (i.e., singular constants or variables). One of the plural predicates, '$\mathbf{H}^2$', which amounts to 'is one of', has a special logical significance. Like the elementary language predicate '=', it is a logical predicate. And we can use it to define complex plural predicates that result from 'expanding' singular predicates. We can define the *plural* (or *neutral*) *expansion* π^N of π as follows:

Def. 1 (neutral expansion):

$$\pi^N(xs) \equiv_{df} \forall y[\mathbf{H}(y, xs) \rightarrow \pi(y)] \text{ (to use the } \lambda \text{ notation, } \pi^N =_{df} \lambda xs\forall y[\mathbf{H}(y, xs) \rightarrow \pi(y)]) .$$

Then the neural expansion 'C^N' of the elementary language counterpart 'C' of 'is a critic', for example, amounts to the predicate 'are critics' (or, more precisely, '*to* be critics), which can be taken to paraphrase '*to* be such that any one of them *is a critic*'.

Now, we can paraphrase (2) into plural languages with 'C' and 'A' as follows:

$$\Sigma xs\{C^N(xs) \wedge \forall x\forall y[\mathbf{H}(x, xs) \wedge A(x, y) \rightarrow x \neq y \wedge \mathbf{H}(y, xs)]\} .^{22} \tag{2*}$$

This amounts to a sentences that we can see is a natural paraphrase of (2): 'Some things are such that they are critics (i.e., any one of them is a critic), and any one of them admires something only if the latter is not the former and is one of them.'[23]

We can now consider sentences with 'most', such as those mentioned above:

Most are funny. (1a)
Most boys are funny. (1b)

These sentences involve plurals, as much as (2) does.[24] But Rescher's symbolic counterpart of (1a), '$MxF(x)$', results from reducing (1a) into a singular construction. The so-called plurality quantifier 'M', like the elementary quantifiers '$\exists$' and '$\forall$', is a *singular quantifier*, one that can combine with singular variables (e.g., 'x'), but not with plural variables (e.g., 'xs').[25] Rescher was working in the framework of the incipient generalized quantifier theory,[26] which adds

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[21] Such argument places are called *plural argument places*. The plural argument places (of plural language predicates) are *neutral* ones, that is, they admit singular terms as well.

[22] That is, $\boldsymbol{\Sigma} xs\{\forall y[\mathbf{H}(y, xs) \rightarrow \mathrm{C}(y)] \wedge \forall x\forall y[\mathbf{H}(x, xs) \wedge A(x, y) \rightarrow x \neq y \wedge \mathbf{H}(y, xs)]\}$.

[23] Using this paraphrase of (2) while analyzing the logic of the expressions involved in (2*), we can explain that (2.1) implies (2) while (2) does not imply the existence of a set of critics.

[24] Note that 'most' is used as the superlative of 'many', rather than of 'much', in (1a)–(1b). The quantifier as so used combines only with plurals.

[25] As a result, Rescher's English reading of '$Ma\phi(a)$' (or '$Mx\phi(x)$') is incoherent and vacillating. He reads it sometimes as "For *most* individuals *a* . . . ϕa" ([12], 373; original italics, my underline), and sometimes as "For most *x's* (of the empty domain D) ϕx" ([12], 374). In the latter sentence, I think, he spontaneously uses '*x's*' as a plural variable. Plurals die hard!

[26] Rescher mentions Mostowski [10], who takes generalized quantifiers to be (singular) second-order predicates that take elementary language predicates as arguments.

quantifiers to elementary languages without changing their underlying singular character. Similarly, the generalized quantifier theory introduces a quantifier that corresponds to the use of ‘most’ in (1b), ‘Q^{most}’, as a binary quantifier that is a par with the elementary language quantifiers except that it takes two elementary language predicates. One can then paraphrase (1b) as follows:

$$Q^{most}x(B(x), F(x)) \ . \tag{1b*}$$

Because it ignores the plural character of ‘most’, the generalized quantifier theory cannot give a proper treatment of siblings of (1a)–(1b) that involve predicates whose analogues cannot be found in elementary languages, such as the following (where ‘Bob’ refers to a huge piano):

Most of the boys lifted Bob. (1c)
Most of the boys who surrounded Bob lifted Bob. (1d)

One cannot paraphrase these into generalized quantifier languages (where ‘b’, ‘L’, and ‘S’ amount to ‘Bob’, ‘lifted’, and ‘surrounded’, respectively) as follows:

$$Q^{most}x(B(x), L(x, b)). \tag{1c*}$$
$$Q^{most}x(S(x, b), L(x, b)). \tag{1d*}$$

(1c*) amounts to ‘Most of the boys individually lifted Bob’ (or ‘Every one of some things that are most of the boys lifted Bob’). But this is not equivalent to (1c), which is true if most of the boys, none of whom can lift Bob, cooperated to lift Bob. Similarly, (1d*) cannot be taken to paraphrase (1d), because it amounts to ‘Most of the boys who *each* surrounds Bob *individually* lifted Bob.’

To deal with this problem, advocates of the generalized quantifier theory might consider building up their languages on plural languages.[27] But those who embrace plurals as peers of singulars need not revert to the generalized quantifier approach to accommodate ‘most’ and the like. They can take a more natural approach unconstrained by its bias for singulars.

In plural languages, we can introduce a one-place plural predicate, ‘$\mathbf{Most}^1$’, and a two-place one, ‘$\mathbf{Most}^2$’, that amount to ‘most’ in (1a) and (1b), respectively. Then ‘$\mathbf{Most}^1(xs)$’ amounts to ‘They *are most* (of all the things)’, and ‘$\mathbf{Most}^2(xs, ys)$’ to ‘The former *are most of* the latter’ (where ‘the former’ and ‘the latter’ are used as anaphoric pronouns).[28] Then we can paraphrase (1a) into plural languages with ‘$\mathbf{Most}^1$’ and the singular predicate ‘F’ as follows:

$$\mathbf{\Sigma}xs[\mathbf{Most}^1(xs) \wedge F^N(xs)], \text{ where ‘}F^N\text{’ is the neutral expansion of ‘}F\text{’}. \tag{1*a}$$

This amounts to ‘There are some things that *are most* (of all the things), and they are funny’, which (1a) can be taken to paraphrase. Similarly, we can paraphrase (1b) as follows:

$$\mathbf{\Sigma}xs\{\forall z[\mathbf{H}(z, xs) \leftrightarrow B(z)] \wedge \mathbf{\Sigma}ys[\mathbf{Most}^2(ys, xs) \wedge F^N(xs)]\} \tag{1*b}$$

which amounts to ‘There are some *things of which anything is one if and only if it is a boy*, and there are some things that are most of the former, and are funny’, which (1b) can be taken to paraphrase.

We can give a similar paraphrase of (1c) into plural languages as follows:

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[27] McKay ([9], Ch. 5) takes this approach.

[28] The 1-place predicate ‘$\mathbf{Most}^1$’ can be defined in terms of the 2-place ‘$\mathbf{Most}^2$’:

$$\mathbf{Most}^1(xs) \equiv_{df} \mathbf{\Sigma}ys[\forall z\mathbf{H}(z, ys) \wedge \mathbf{Most}^2(xs, ys)].$$

$$\Sigma xs\{\forall z[\mathrm{H}(z, xs) \leftrightarrow B(z)] \wedge \Sigma ys[\mathbf{Most}^2(ys, xs) \wedge \boldsymbol{L}(ys, b)]\} \qquad (1^*c)$$

where '**L**' is a two-place plural predicate that amounts to 'surrounded' in, e.g., 'They surrounded Bob'. (1*c) differs from (1*b) in an important respect: while (1*c) has this plural predicate, (1*b) has no undefined plural predicate (except '$\mathbf{Most}^2$', which corresponds to 'Q^{most}'). So (1c) (or, equivalent, (1*c)), unlike (1b) (or, equivalently, (1*b)), cannot be paraphrased into the usual generalized quantifier languages, just as 'Some boys lifted Bob', unlike 'Some boys are funny', cannot be paraphrased into elementary languages.

Note that (1b*) and (1c*) amount to 'Most of *the boys* lifted Bob' and 'Most of *the boys* are funny', respectively, because 'They are (*all*) *the boys*' can be taken to paraphrase 'They are some *things of which anything is one if and only if it is a boy*.' So it is useful to introduce symbolic counterparts of the plural definite description 'the boys' and the like. Let '$<x: B(x)>$' be the symbolic counterpart of 'the boys'. Then we can use it to abbreviate '$\forall z[\mathrm{H}(z, xs) \leftrightarrow B(z)]$' in (1*b) and (1*c) as follows:[29]

$$\Sigma ys[\mathbf{Most}^2(ys, <x: Bx>) \wedge \boldsymbol{L}(ys, b)] \text{ (or, } \lambda xs\{\Sigma ys[\mathbf{Most}^2(ys, xs) \wedge \boldsymbol{L}(ys, b)]\}(<x: Bx>)). \qquad (1^*b^*)$$
$$\Sigma ys[\mathbf{Most}^2(ys, <x: Bx>) \wedge \boldsymbol{L}(ys, b)] \text{ (or, } \lambda xs\{\Sigma ys[\mathbf{Most}^2(ys, xs) \wedge \boldsymbol{L}(ys, b)]\}(<x: Bx>)). \qquad (1^*c^*)$$

Now, because some things are, e.g., the boys if and only if they are *things of which anything is one if and only if it is a boy*, we can give a contextual definition of the definite description '$<x: B(x)>$' and the like in plural languages as follows:

Def. 2 (*Plural definite descriptions of the first kind*)

$$\pi(<x: \phi(x)>) \equiv_{df} \Sigma xs\{\forall z[\mathrm{H}(z, xs) \leftrightarrow \phi(z)] \wedge \pi(xs)\}, \text{ where } \pi \text{ is a predicate.}$$

Applying this to (1*b*) and (1*c*) yields (1*b) and (1*c), respectively.[30, 31]

The plural definite description in (1d), 'the boys who surrounded Bob', requires a different treatment. It cannot be analyzed in the same way as, e.g., 'the boys'. Compare the following:

Something is one of the boys if and only if it is a boy. (a)
Something is one of the boys who surrounded Bob if and only if it is a boy who surrounded Bob. (b)

Although (a) is a logical truth, which provides the basis for *Def.* 2, (2) might be false. So it is necessary to give a different analysis of (1d) to paraphrase it into plural languages. Now, we can introduce into plural languages plural

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[29] In both (1*b*) and (1*c*), '$<x: Bx>$' takes the widest scope.

[30] Using the definite description, we can give a simple formulation of (1*a): '$\mathbf{Most}^1(<x: F(x)>)$.' Applying *Def.* 2 (together with *Def.* 1) to this yields a straightforward logical equivalent of (1*a).

[31] (1*b) (or, equivalently, (1*b*)) implies the existence of a boy. So 'Every boy is funny', which does not imply the existence of a boy, does not imply 'Most boys are funny' on my analysis. The generalized quantifier analysis of 'most' yields the same result. Moreover, the usual generalized quantifier account considers 'all' in, e.g., 'All boys are funny' or 'All the boys are funny' a mere variant of 'every' to take these sentences to be equivalent to 'Every boy is funny' so that they also fail to imply (1b). But we can treat 'all', which combines only with plural forms of count nouns, like 'most'. In plural languages, we can introduce a two-place plural predicate, '$\mathbf{All}^2$', that amounts to 'are *all* of' in, e.g., 'They *are all of* my friends.' We can then take 'All boys are funny' to be equivalent to 'All the boys are funny', and paraphrase it by '$\Sigma xs[\mathbf{All}(xs, <x: Bx>) \wedge F^N(xs)]$'. And we can define '$\mathbf{All}^2$' using only logical expressions of plural languages as follows:

$$\mathbf{All}^2(xs, ys) \equiv_{df} \forall z[\mathrm{H}(z, xs) \leftrightarrow \mathrm{H}(z, ys)] \text{ (or } \mathbf{All}^2 =_{df} \lambda xs,ys\forall z[\mathrm{H}(z, xs) \leftrightarrow \mathrm{H}(z, ys)]) .$$

For some things are *all* of, e.g., my friends if and only if any one of them is one of my friends, and *vice versa*. We can then show that '$\Sigma xs[\mathbf{All}(xs, <x: Bx>) \wedge F^N(xs)]$' is logically equivalent to '$\exists xB(x) \wedge \forall x[B(x) \rightarrow F(x)]$', and implies (1*b) (for '$\Pi xs.\mathbf{Most}(xs, xs)$' or 'Any things are most of themselves holds).

definite descriptions of another kind, those that amount to 'the boys who surrounded Bob' and the like. Let '(Izs)$\boldsymbol{S}$(zs, b)', for example, be a definite description that amounts to this. Using this, we can paraphrase (1d) as follows:

$$\Sigma ys[\mathbf{Most}^2(ys, (\mathrm{I}xs)\boldsymbol{S}(xs, b)) \wedge \boldsymbol{L}(ys, b)] \text{ (or, } \lambda xs\{\Sigma ys[\mathbf{Most}^2(ys, xs) \wedge \boldsymbol{L}(ys, b)]\}((\mathrm{I}xs)\boldsymbol{S}(xs, b)) . \qquad (1^*\text{d})$$

This, which results from replacing '<x: Bx>' in (1*c*) with '(Ixs)$\boldsymbol{S}$(xs, b)', amounts to 'There are some things that are most of *the boys who surrounded Bob*, and they lifted Bob.' Now, one can give a contextual definite of plural definite descriptions of the second kind as well in plural languages. Here is the definition as applied to '(Ixs)$\boldsymbol{S}$(xs, b)':[32]

Def. 3 (*Plural definite descriptions of the second kind* [an example])
$$\pi((\mathrm{I}xs)\boldsymbol{S}(xs, b)) \equiv_{\mathrm{df}} \Sigma xs\{\forall zs[\boldsymbol{S}(zs, b) \leftrightarrow zs \approx xs)] \wedge \pi(xs)\}, \text{ where } \pi \text{ is a predicate.}$$

Here '$zs \approx xs$' abbreviates '$\forall y[\mathrm{H}(y, zs) \leftrightarrow \mathrm{H}(y, xs)]$' ('$\approx$' can be used to paraphrase '*to* be the same things as'). Applying the definition to (1*d) yield the following:

$$\Sigma xs\{\forall zs[\boldsymbol{S}(zs, b) \leftrightarrow zs \approx xs)] \wedge \Sigma ys[\mathbf{Most}^2(ys, xs) \wedge \boldsymbol{L}(ys, b)]\} . \qquad (1^*\text{d}^*)$$

This amounts to, roughly, 'There are some things that lifted Bob, and they are most of *the things that are the same as some things if and only if these surrounded Bob*.'

Now, note that the plural language analysis of 'most' *explains* the fact that 'most' is a so-called *conservative* quantifier: (1b) is equivalent to 'Most of the boys *are boys and* are funny', because any things that are most of the boys must be boys. Although the generalized quantifier attributes conservativity to 'most', however, its conservativity has an important limitation in scope: (1d) is not equivalent to 'Most of the boys who surrounded Bob *are boys who surrounded Bob and* lifted Bob.' The plural language analysis of 'most' explains this limitation as well: those who are most of *the boys who surrounded* (*or lift*) *Bob* might not themselves suffice to *surround* (*or lift*) *it*. Proponents of the generalized quantifier theory fail to note the limitation, let alone explain it, because they in effect begin by placing, e.g., (1d) beyond the scope of the theory because it cannot be put in their favorite languages. But it would be wrong to hold that 'most' is used differently in (1b) or (1c) and (1d). My analysis of the quantifier, as we have seen, explains the logical difference between the sentences as arising from the difference in the logical character between two kinds of definite descriptions.

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[32] See Yi ([20], *sc*. 4) for the definition and general discussions of plural definite descriptions.

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Appendix

We can use some basic results of finite model theory to show the following:

(A) There is no elementary language sentence that agrees with Rescher's "plurality" quantification '$MxF(x)$' on all finite models.

(B) There is no elementary sentence that agrees with the Geach-Kaplan sentence (2), 'Some critics admire only one another', on all finite domains.

Let $\mathfrak{L}_F$ be the elementary language whose only non-logical expression is 'F^1', and $\mathfrak{L}_A$ the elementary language whose only non-logical expression is 'A^2'. Then a model $\mathbf{M}$ of $\mathfrak{L}_F$ is a pair $<\mathbf{D^M}, F^{\mathbf{M}}>$ such that $\mathbf{D^M}$ is a non-empty set and $F^{\mathbf{M}}$ a subset of $\mathbf{D^M}$; and a model $\mathbf{M}$ of $\mathfrak{L}_A$ a pair $<\mathbf{D^M}, A^{\mathbf{M}}>$ such that $\mathbf{D^M}$ is a non-empty set and $A^{\mathbf{M}}$ a subset of $\mathbf{D^M}\times\mathbf{D^M}$. Say that a model $\mathbf{M}$ is *finite*, if its domain, $\mathbf{D^M}$, is a finite set. Then we can show the following:[33]

(A*) There is no sentence ϕ of $\mathfrak{L}_F$ such that any finite model $\mathbf{M}$ of $\mathfrak{L}_F$ satisfies ϕ if and only if $|\mathbf{D^M}\backslash F^{\mathbf{M}}| = |F^{\mathbf{M}}|$.

But '$-MxF(x) \wedge -Mx-F(x)$' is such a sentence. So (A) holds. To state a theorem that we can use to show (B), it is useful to use the following notions:

Definitions: Let $\mathbf{M}$ ($=<\mathbf{D^M}, A^{\mathbf{M}}>$) be a model of $\mathfrak{L}_A$. Then

1. $\mathbf{M}$ is a *graph*, if for any members a and b of $\mathbf{D^M}$, not $A^{\mathbf{M}}(a, a)$, and if $A^{\mathbf{M}}(a, b)$, then $A^{\mathbf{M}}(b, a)$.
2. There is *a path* between a and b, if a and b are members of $\mathbf{D^M}$ and either $A^{\mathbf{M}}(a, b)$ or there are finitely many members $x_1, x_2, ..., x_{n-1}, x_n$ of $\mathbf{D^M}$ such that $A^{\mathbf{M}}(a, x_1)$, $A^{\mathbf{M}}(x_1, x_2)$, ..., $A^{\mathbf{M}}(x_{n-1}, x_n)$, $A^{\mathbf{M}}(x_n, y)$.
3. A graph is *connected*, if there is a path between any two members of $\mathbf{D^M}$.

[33] This is a variant of the theorem of undefinability of the class of even-numbered models (among finite models). For a proof of the theorem, see, e.g., Väänänen ([14], 6). We can use compactness of elementary logic to show (A*).

Then the following is a theorem of finite model theory:[34]

(B*) There is no sentence ϕ of $\mathfrak{L}_A$ such that any finite model **M** of $\mathfrak{L}_A$ satisfies ϕ if and only if **M** is a connected graph.

Now, if there is an elementary language sentence that agrees with (2) on all finite models, then there is an elementary language sentence that agrees with the following on all finite models:

There is something such that there are some things that are not identical with it that admire only one another.

(And we may assume such a sentence is a sentence of $\mathfrak{L}_A$.) So let ϕ be a sentence of $\mathfrak{L}_A$ that agrees with the above sentence on all finite models. Then let ϕ^* be $[\forall x{-}A(x, x) \wedge \forall x \forall y(A(x, y) \leftrightarrow A(y, x)) \wedge \phi]$. Then a finite model **M** of $\mathfrak{L}_A$ satisfies ϕ^* if and only if **M** is a connected graph, which violates (B*). So (B) holds.

[34] See, e.g., Ebbinghaus & Flum ([4], 22f) or Väänänen ([14], 9) for a proof.