Hyperintensional Dynamic Semantics: Analyzing Definiteness with Enriched Contexts*

Scott Martin and Carl Pollard

Department of Linguistics
Ohio State University
1712 Neil Avenue
Columbus, Ohio 43210 USA
{scott,pollard}@ling.ohio-state.edu

Abstract. We present a dynamic semantic theory that synthesizes aspects of de Groote's (2006) continuation-based dynamics and Pollard's (2008a) hyperintensional semantics. In this theory, we rely on an enriched notion of discourse context inspired by the work of Heim (1983a,b) and Roberts (1996, 2004). We show how to use this enriched context to improve on de Groote's treatment of English definite anaphora.

1 Introduction

As Muskens (1994, 1996) showed, many of the insights of dynamic semantic theories such as discourse representation theory (DRT, Kamp, 1981; Kamp and Reyle, 1993) and file change semantics (FCS, Heim, 1982, 1983b) can be formalized within the well-understood framework of classical higher order logic (HOL, Church, 1940; Henkin, 1950; Gallin, 1975) familiar to Montague semanticists. Also working within HOL, de Groote (2006) showed that the description of context update could be streamlined by modelling right contexts by analogy with the *continuations* employed in programming language semantics (Strachey and Wadsworth, 1974).

Though both Muskens' and de Groote's work are positive developments in the sense of helping to integrate dynamic notions into mainstream semantic theory, both fall short in modelling how definite anaphora works in discourse, which was one of the two central problems that Kamp and Heim originally set out to solve. (The other was to characterize the novelty of indefinite descriptions.) For Muskens, definite pronouns are simply ambiguous with respect to which accessible and sortally appropriate discourse referent they 'pick up'. The trouble with this theory is that, empirically, definite anaphora is generally *not* ambiguous; if it were, it would fail to serve its communicative purpose.

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De Groote fares no better. On his account, the antecedent of a pronoun is picked out by an oracular choice function `sel` from among the sortally correct candidate entities (de Groote does not have discourse referents as distinct from entities in his theory). At first one might think the weakness of this account is that it doesn't tell us anything about *what* choice function this oracle actually is. But in fact it is worse than that, because it is easy to show that *no* choice function is the right one. That is because, in general, the antecedent of a definite pronoun fails to be uniquely determined by the set of sortally appropriate candidate entities. Consider, e.g., the following two narratives:

(A)
1. A donkey and a mule walked in.
2. The donkey was sad.
3. It brayed.

(B)
1. A donkey and a mule walked in.
2. The mule was sad.
3. It brayed.

In both (A) and (B), the set of candidate antecedents consists of two nonhuman entities, a donkey and a mule. In each narrative, one of the candidates has been rendered more salient by virtue of having been re-invoked by a definite description. And in each narrative, the anaphora resolves to the more salient, 'definitized' entity. But the choice function only 'knows about' the members of the set of candidates and their sortal properties, not about their relative salience in the discourse at hand. So it cannot pick the right antecedent both times.

In our view, the weaknesses of Muskens' and de Groote's theories arise because they fail to build in a notion of context that is sufficiently rich to support a satisfactory account of *presuppositions*, the conditions on contexts that must be satisfied in order for utterances to be felicitous. In this paper, we suggest a revision and extension of de Groote's theory that copes with definite anaphora by building in a (slightly) more articulated discourse model inspired by proposals due to Roberts (1996). However, the work reported in this paper is part of a larger research program, joint with Roberts and Smith, aimed at constructing formally explicit, categorially-based, natural-language grammars that deal effectively with projective aspects of meaning (Roberts et al., 2009). Besides Roberts' work on modelling discourse contexts, this research program also builds on Pollard (2008a,b)'s hyperintensional semantics (relevant aspects of which are sketched below), and on 'pheno-tecto grammar' (PTG, Oehrle, 1994; de Groote, 2001; Muskens, 2007; Smith, 2010), the line of development in categorial grammar that distinguishes concrete syntax from combinatorics (not touched upon in this paper).

The rest of this paper is organized as follows. We present some facts about English definiteness in Sect. 2. In Sect. 3, we formally lay out our hyperintensional dynamic semantic theory of discourse. Specific machinery for dealing with definiteness is introduced in Sect. 4 that handles some of the cases introduced in Sect. 2. Section 5 concludes.

2 Facts about Definiteness in English

The use of certain English expressions, such as proper names, the definite article *the*, and pronouns such as *it*, carries associated presuppositions about the discourse context. These presuppositions have to do with **definiteness**, which encompasses—in senses to be made precise—salience, familiarity, and uniqueness. To take the most simple example, if

(C) # It brayed

is uttered out of the blue and in the absence of some salient perceptible nonhuman entity in the immediate surroundings that might plausibly have brayed, the presupposition of the pronoun is not fulfilled and so the utterance is infelicitous. Such a salience presupposition need not be globally satisfied, but instead can be satisfied locally (roughly: within the scope of one or more operators) as in

(D) No donkey denies it brays.

For this reason the salience presuppositions associated with definite anaphora are usually taken to have to do with the availability not of entities *per se* but rather of "discourse referents", a notion which the ambient theory must make precise.

Definites also presuppose the familiarity of the intended discourse referent:

(E)
1. I saw the donkey.
2. What donkey?
3. # Oh, just some donkey out in a field on the way to Upper Sandusky.

In (E), even in the presence of one or more donkeys, the use of the definite article is infelicitous unless there is one that has been made familiar by virtue of having been introduced into the discourse context.

In addition to being salient and familiar, both *it* and *the* also presuppose a discourse referent that is contextually unique (uniquely maximally salient). The discourses in (F) and (G) demonstrate uniqueness presuppositions:

(F)
1. A donkey had a red blanket.
2. A mule had a blue blanket.
3. $\left\{\begin{array}{l}\text{The donkey}\\ \text{\# It}\end{array}\right\}$ snorted.

(G)
1. A donkey had a red blanket.
2. Another donkey had a blue blanket.
3. $\left\{\begin{array}{l}\text{The donkey with the blue blanket}\\ \text{\# The donkey}\\ \text{\# It}\end{array}\right\}$ snorted.

In (F), salience alone is not enough to determine whether *a donkey* or *a mule* antecedes the pronoun *it*. The identity of the antecedent must be uniquely determinable from the discourse context, but since *it* can be anteceded by any nonhuman entity in English, the noun phrase *the donkey* is used instead to unambiguously single out the pronoun's unique antecedent. The discourse in (G) is a variant of (F) where the property *donkey* is not enough to disambiguate the antecedent because of the uniqueness presuppositions associated with *the* and *it*.

Finally, the absurd discourse in (H) shows that the definite article identifies not just a maximally salient individual, but one with a certain specified property:

(H) 1. I saw the donkey.
2. What donkey?
3. # That llama we always see on the way to Findlay.

Here, it is infelicitous to use the noun phrase *the donkey* to identify the llama even if it is the most salient individual in the utterance context because it does not have the property of being a donkey.

3 Hyperintensional Dynamic Semantics

Our formalization of discourse dynamics builds on the hyperintensional theory of (static) meaning in Pollard (2008a). Like Montague semantics (Montague, 1973), this semantic theory is couched in HOL and has a basic type e for entities as well as the truth-value type t provided by the underlying logic.[1]

But unlike Montague semantics, we follow Thomason (1980) in assuming a basic type p for (static) propositions (but no basic type for worlds).[2] Following Lambek and Scott (1986), we also assume (1) a natural number type; (2) the type constructors U (unit type) and $\times$ (cartesian product) in addition to the usual $\rightarrow$ (exponential); and (3) separation-style subtyping.[3] Subtypes are usually written in the form $\{x \in T \mid \varphi[x]\}$ where $\varphi[x]$ is a formula (boolean term) possibly with x free. Additionally, we make use of dependent coproduct types parameterized by the natural number type, written $\amalg_n T_n$.

From the typed lambda calculus that underlies the HOL, we have the usual pairing and projection functions; the function application $(\mathsf{app}\, f\, a)$ is abbreviated $(f\ a)$ rather than $f(a)$.

The type of propositions is axiomatized as a preboolean algebra (like a boolean algebra, but without antisymmetry), preordered by entailment. The propositional connectives and quantifiers are written as boldface versions of the

[1] For expository simplicity, we depart from Pollard (2008a) in not distinguishing between the extensional type e and the corresponding hyperintensional type i (individual concepts).

[2] Using separation subtyping, we can define the type of worlds as a certain subtype of the type p $\rightarrow$ t (sets of propositions), but this will not be needed here.

[3] Thus if A is a type and a an A-predicate (term of type $A \rightarrow$ t), then there is a type A_a interpreted as the subset of the interpretation of A that has the interpretation of a as its characteristic function; and there is a constant μ_a that denotes the subset embedding.

usual (boolean) connectives of the underlying logic: e.g. $\neg$, $\wedge$, $\vee$, $\longrightarrow$, $\exists$, and $\forall$; true is a (certain) necessarily true proposition.

Following Heim, we use natural numbers (type ω) as discourse referents (hereafter, DRs). The type ω is equipped with the usual linear order $<$ and the successor function $\mathbf{suc} : \omega \to \omega$. Additionally, for each natural number n, we define the type of the first n natural numbers as a subtype of ω:

$$\omega_n =_{\text{def}} \{i \in \omega \mid i < n\} \tag{1}$$

These types will be used for the domains of assignment functions.

We adopt the convention that applications and pairings associate to the left and abstractions associate to the right. Parentheses are sometimes abbreviated using . in the usual way (e.g., $\lambda_x.M$) or omitted altogether when no confusion can arise. When a term contains multiple embedded λ-abstractions of the form $\lambda_a\lambda_b\lambda_c M$, we collapse them together as $\lambda_{abc}M$.

3.1 Information Structures

To advance from static to dynamic semantics, we need to extend our ontology to model contexts. Our notion of context is simplified version of Roberts' (1996) *discourse information structures*, here called simply *structures*.[4] A structure is a tuple consisting of (1) an *assignment* of entities to a set of DR's, (2) a salience preorder on those DR's called the *resolution* preorder (so-called because it will be used to resolve definite anaphora), and (3) a proposition, the *common ground*, which is the conjunction of all the propositions that are taken by the interlocutors to be mutually agreed upon. The common ground includes not only propositions explicitly asserted and accepted in the discourse, but also encyclopedic knowledge about the world that is assumed as shared background.

To make this notion of structure more precise, we begin by defining the type of n-ary assignments functions α_n to be the type of functions from the first n discourse referents to entities:

$$\alpha_n =_{\text{def}} \omega_n \to \mathrm{e} \tag{2}$$

$$\alpha =_{\text{def}} \amalg_n \alpha_n \tag{3}$$

The length function $\mathbf{l} : \alpha \to \omega$ gives the length (size of the domain) of an n-ary assignment, via the axiom

$$\vdash \forall_{a:\alpha_n}(\mathbf{l}\, a) = n \tag{4}$$

In dynamic interpretations, an assignment function's length is used as the "next" DR. An n-ary assignment can be extended to include a new DR mapped to a specified entity using the function $\bullet : \alpha_n \to \mathrm{e} \to \alpha_{(\mathbf{suc}\, n)}$ (written infix), that is subject to the axiom schema

$$\vdash \forall_{a:\alpha_n}\forall_{x:e}\forall_{m:\omega_{(\mathbf{suc}\, n)}}(a \bullet x)\, m = \begin{cases} x & \text{if } m = n \\ (a\, m) & \text{otherwise} \end{cases} \tag{5}$$

[4] At this stage, we omit Roberts' moves, domain goals, QUD stack, etc.

To track the relative salience of the DRs in the domain of an assignment, we use a preorder of the same arity. For arbitrary $n : \omega$, an n-ary *resolution* is just a preorder (reflexive, transitive relation) on the set of DRs:

$$\rho_n =_{\mathrm{def}} \{r \in \omega_n \rightarrow \omega_n \rightarrow \mathrm{t} \mid r \text{ is a preorder}\} \tag{6}$$

Note that this is a subtype of the type of binary relations on ω_n; here "r is a preorder" abbreviates a formula which says of the binary relation r that it *is* a preorder. Below, we will see that DRs which are "higher" in the resolution preorder are "better" candidates for subsequent definite anaphora.

The function $\star : \rho_n \rightarrow \rho_{(\mathbf{suc}\, n)}$ is used to extend a resolution to the next larger domain, subject to the following schema:

$$\vdash \forall_{r:\rho_n} n\, (\star\, r)\, n \tag{7}$$

Resolution extension thus occurs in a noncommittal way: for a resolution $r : \rho_n$, the extended resolution $(\star\, r)$ has n reflexively as high as itself, but leaves n incomparable to every $m < n$.

Information structures combine an n-ary assignment and resolution with a proposition, the common ground of the discourse. The type σ_n, mnemonic for *structure*, is a triple defined as:

$$\sigma_n =_{\mathrm{def}} \alpha_n \times \rho_n \times \mathrm{p} \tag{8}$$

$$\sigma =_{\mathrm{def}} \amalg_n \sigma_n \tag{9}$$

Similar to the type α for assignments (see (2) and (3), above), the type σ combines all the types σ_n together as a single type. The type σ plays a role analogous to that of γ (left contexts) in de Groote's (2006) type-theoretic dynamics, but enriches his notion of discourse context to include salience and a common ground in addition to a set of DRs.

The functions $\mathbf{a} : \sigma \rightarrow \alpha$ (for *assignment*), $\mathbf{r} : \sigma \rightarrow \rho$ (for *resolution*) and $\mathbf{c} : \sigma \rightarrow \mathrm{p}$ (for *common ground*) are just the projections from σ to its three components. These functions are used to access the components of a structure.

To extend a structure with a new entity (i.e., introduce a new discourse referent and assign a certain value to it), we use the function $\mathbf{intro} : \sigma \rightarrow \mathrm{e} \rightarrow \sigma$:

$$\vdash \mathbf{intro} = \lambda_{sx} \langle \mathbf{a}\, s \bullet x, \star\, (\mathbf{r}\, s), \mathbf{c}\, s \rangle \tag{10}$$

This enriched replacement for de Groote's :: extends both a structure's assignment and its resolution. The function $\mathbf{upd} : \sigma \rightarrow \mathrm{p} \rightarrow \sigma$ adds the ability to update the common ground of a structure with a new proposition:

$$\vdash \mathbf{upd} = \lambda_{sp} \langle \mathbf{a}\, s, \mathbf{r}\, s, (\mathbf{c}\, s) \wedge p \rangle \tag{11}$$

where $\wedge$ is propositional, not boolean, conjunction.

3.2 Continuations and Dynamic Semantics

The type κ of continuations is the type of functions from structures to propositions:

$$\kappa =_{\text{def}} \sigma \to \text{p} \tag{12}$$

Modulo replacement of de Groote's γ (left contexts) and o (truth values) by σ and p respectively, our continuations are direct analogs of his right contexts ($\gamma \to o$). The **null continuation** is $\lambda_s \mathsf{true}$, where true is a necessarily true proposition.

A **dynamic proposition** (type π) maps a structure and a continuation to a (static) proposition:

$$\pi =_{\text{def}} \sigma \to \kappa \to \text{p} \tag{13}$$

This is a direct analog of de Groote's type Ω. Extending Muskens (1994, 1996), for each $n \in \omega$, we recursively define the type of n-ary **dynamic relations** as follows:

$$\delta_0 =_{\text{def}} \pi \tag{14}$$

$$\delta_{n+1} =_{\text{def}} \omega \to \delta_n \tag{15}$$

We abbreviate δ_1, the type of dynamic properties, as simply δ.

Instances of dynamic propositions and properties are given in the following axioms:

$$\vdash \textsc{rain} = \lambda_{sk}.\mathsf{rain} \wedge k\ (\mathbf{upd}\ s\ \mathsf{rain}) : \pi \tag{16}$$

$$\vdash \textsc{snow} = \lambda_{sk}.\mathsf{snow} \wedge k\ (\mathbf{upd}\ s\ \mathsf{snow}) : \pi \tag{17}$$

$$\vdash \textsc{donkey} = \lambda_{nsk}.(\mathsf{donkey}\ (\mathbf{a}\ s)\ n) \wedge k\ (\mathbf{upd}\ s\ (\mathsf{donkey}\ (\mathbf{a}\ s)\ n)) : \delta \tag{18}$$

$$\vdash \textsc{bray} = \lambda_{nsk}.(\mathsf{bray}\ (\mathbf{a}\ s)\ n) \wedge k\ (\mathbf{upd}\ s\ (\mathsf{bray}\ (\mathbf{a}\ s)\ n)) : \delta \tag{19}$$

These examples show how dynamic propositions/properties interact with the structure of the utterance context they are situated inside. The common ground is always updated via **upd** with the proffered content (cf. Roberts, 1996) and passed to the rest of the discourse (in the form of the continuation k). In the case of the dynamic properties DONKEY and BRAY, these expect an argument that is not an entity but instead a natural number (i.e., DR) which is mapped to an entity by the assignment ($\mathbf{a}\ s$).

The static propositional content of a dynamic proposition in context is retrieved using **cont** : $\sigma \to \pi \to \text{p}$ (mnemonic for *proffered* or *contributed content*), a direct analog of de Groote's (2008) READ:

$$\vdash \mathbf{cont} = \lambda_{sA}.A\ s\ \lambda_s \mathsf{true} \tag{20}$$

This function gives the dynamic proposition A access to the context s, but then "throws away" the rest of the discourse by specifying the null continuation as its κ-type argument.

Example 1 (Proffered Content of a Dynamic Proposition). Assuming that the context $s : \sigma$ is such that $(\mathbf{a}\, s)\, n = x$ for some $n : \omega$, we calculate the proffered content of $(\textsc{donkey}\, n)$ as follows:

$$\begin{aligned}
\mathbf{cont}\, s\, (\textsc{donkey}\, n) &= \mathbf{cont}\, s\, \lambda_{sk}.(\textsf{donkey}\, (\mathbf{a}\, s)\, n) \wedge k\, (\mathbf{upd}\, s\, (\textsf{donkey}\, (\mathbf{a}\, s)\, n)) \\
&= \mathbf{cont}\, s\, \lambda_{sk}.\textsf{donkey}\, x \wedge k\, (\mathbf{upd}\, s\, (\textsf{donkey}\, x)) \\
&= (\lambda_{sk}.\textsf{donkey}\, x \wedge k\, (\mathbf{upd}\, s\, (\textsf{donkey}\, x)))\, s\, \lambda_s \textsf{true} \\
&= (\lambda_{k}.\textsf{donkey}\, x \wedge k\, (\mathbf{upd}\, s\, (\textsf{donkey}\, x)))\, \lambda_s \textsf{true} \\
&= \textsf{donkey}\, x \wedge (\lambda_s \textsf{true}\, (\mathbf{upd}\, s\, (\textsf{donkey}\, x))) \\
&= \textsf{donkey}\, x \wedge \textsf{true} \\
&\equiv \textsf{donkey}\, x
\end{aligned}$$

where DONKEY is as defined in (18) and $\equiv$ denotes propositional equivalence (cf. Pollard, 2008a).

Dynamic Connectives The dynamic negation $\textsc{not} : \pi \to \pi$ is similar to de Groote's:

$$\vdash \textsc{not} = \lambda_{Ask}.(\neg\, \mathbf{cont}\, s\, A) \wedge k\, s \tag{21}$$

The null continuation freezes the scope of negation, but the occurrence of s in the scope of the static negation makes NOT a hole for projecting presuppositions, since they possibly depend on the resolution preorder and the common ground.

For dynamic conjunction, we follow de Groote in defining dynamic $\textsc{and} : \pi \to \pi \to \pi$ to compose the meanings of two dynamic propositions over a structure and discourse continuation:

$$\vdash \textsc{and} = \lambda_{ABsk}.A\, s\, (\lambda_s.B\, s\, k) \tag{22}$$

The continuation passed to the first conjunct A is the second conjunct B with its structure (type σ) argument abstracted over. Example 2 demonstrates dynamic conjunction in action.

Example 2 (Dynamic Conjunction). With RAIN and SNOW as defined in (16) and (17) and AND as in (22), the conjunction of RAIN and SNOW is:

$$\begin{aligned}
&\vdash \textsc{rain and snow} : \pi \\
&= \lambda_{sk}.\textsc{rain}\, s\, (\lambda_s.\textsc{snow}\, s\, k) \\
&= \lambda_{sk}(\lambda_{sk}(\textsf{rain} \wedge k\, (\mathbf{upd}\, s\, \textsf{rain}))\, s\, (\lambda_s.\textsc{snow}\, s\, k)) \\
&= \lambda_{sk}(\lambda_{k}(\textsf{rain} \wedge k\, (\mathbf{upd}\, s\, \textsf{rain}))\, (\lambda_s.\textsc{snow}\, s\, k)) \\
&= \lambda_{sk}.\textsf{rain} \wedge (\lambda_s(\textsc{snow}\, s\, k)\, (\mathbf{upd}\, s\, \textsf{rain})) \\
&= \lambda_{sk}.\textsf{rain} \wedge (\lambda_{sk}(\textsf{snow} \wedge k\, (\mathbf{upd}\, s\, \textsf{snow}))\, (\mathbf{upd}\, s\, \textsf{rain})\, k) \\
&= \lambda_{sk}.\textsf{rain} \wedge (\lambda_{k}(\textsf{snow} \wedge k\, (\mathbf{upd}\, (\mathbf{upd}\, s\, \textsf{rain})\, \textsf{snow}))\, k) \\
&= \lambda_{sk}.\textsf{rain} \wedge \textsf{snow} \wedge k\, (\mathbf{upd}\, (\mathbf{upd}\, s\, \textsf{rain})\, \textsf{snow})
\end{aligned}$$

Note that the content proffered by RAIN is available in the common ground of the structure passed to SNOW.

This definition of AND plays a central role in our dynamic indefinite, given in (25), below.

As for de Groote, dynamic implication $\Rightarrow : \pi \to \pi \to \pi$ is defined based on de Morgan's laws in terms of NOT and AND:

$$\vdash \Rightarrow = \lambda_{AB}.\textsc{not}\,(A \textsc{ and } (\textsc{not}\, B)) \tag{23}$$

There are two motivations for this definition of $\Rightarrow$. The first is that NOT restricts the scope of DRs introduced inside the conditional. Second, AND makes the structure updated by the antecedent A available as input to the consequent B. This definition of $\Rightarrow$ shows how NOT and AND interact to provide our theory's counterpart of DRT's accessibility relation. The dynamic meaning of *every* (see (27), below) is based on $\Rightarrow$.

Dynamic Quantifiers Our replacement for de Groote's Σ is EXISTS : $\delta \to \pi$:

$$\vdash \textsc{exists} = \lambda_{Dsk}.\exists\, \lambda_x.D\,(\mathbf{l}\,(\mathbf{a}\,s))\,(\mathbf{intro}\,s\,x)\,k \tag{24}$$

This version of the dynamic existential quantifier introduces a DR, using **intro** to extend both the assignment and resolution of the current structure. By taking the length of the current assignment ($\mathbf{l}\,(\mathbf{a}\,s)$), the DR that the assignment extended by **intro** maps to x is then passed as an argument to D along with the newly extended structure and discourse continuation k. The newly introduced entity x is then abstracted over, with the resulting term passed to the propositional quantifier $\exists$.

We use EXISTS and AND to model the dynamic indefinite article A : $\delta \to \delta \to \pi$ as follows:

$$\vdash \textsc{a} = \lambda_{DE}.\textsc{exists}\ \lambda_n.D\,n \textsc{ and } E\,n \tag{25}$$

This definition ensures, via AND, that the scope inherits whatever extensions are made to the structure by the restriction.

Example 3 (Dynamic Indefinite Generalized Quantifier). The behavior of the indefinite article A is illustrated by applying it to DONKEY (as defined in (18)) to yield:

$$\begin{aligned}
&\vdash \textsc{a donkey} : \delta \to \pi \\
&= \lambda_E.\textsc{exists}\ \lambda_n.\textsc{donkey}\,n \textsc{ and } E\,n \\
&= \lambda_{Esk}.\exists\,\lambda_x.(\lambda_n(\textsc{donkey}\,n \textsc{ and } E\,n)\,(\mathbf{l}\,(\mathbf{a}\,s)))\,(\mathbf{intro}\,s\,x)\,k \\
&= \lambda_{Esk}.\exists\,\lambda_x.(\textsc{donkey}\,(\mathbf{l}\,(\mathbf{a}\,s)) \textsc{ and } E\,(\mathbf{l}\,(\mathbf{a}\,s)))\,(\mathbf{intro}\,s\,x)\,k \\
&= \lambda_{Esk}.\exists\,\lambda_x.\mathsf{donkey}\,x \wedge E\,(\mathbf{l}\,(\mathbf{a}\,s))\,(\mathbf{upd}\,(\mathbf{intro}\,s\,x)\,(\mathsf{donkey}\,x))\,k \\
&= \lambda_{Esk}.\exists\,\lambda_x.\mathsf{donkey}\,x \wedge E\,(\mathbf{l}\,(\mathbf{a}\,s))\,\langle \mathbf{a}\,s \bullet x, \star\,(\mathbf{r}\,s), \mathbf{c}\,s \wedge \mathsf{donkey}\,x\rangle\ k
\end{aligned}$$

Note that the sortal restriction imposed by the noun on the new DR is part of the common ground passed to the scope.

The dynamic universal quantifier FORALL : $\delta \to \pi$, which de Groote writes Π, is defined as the de Morgan dual of EXISTS:

$$\vdash \text{FORALL} = \lambda_D.\text{NOT}\,(\text{EXISTS}\,\lambda_n.\text{NOT}\,(D\,n)) \tag{26}$$

Finally, we define dynamic EVERY : $\delta \to \delta \to \pi$ based on FORALL and $\Rightarrow$:

$$\vdash \text{EVERY} = \lambda_{DE}.\text{FORALL}\,\lambda_n.D\,n \Rightarrow E\,n \tag{27}$$

As for propositional generalized quantifiers, this definition makes EVERY a universally quantified conditional relation between two properties. But here, the use of NOT and AND in the definitions of $\Rightarrow$ and FORALL make modifications to the structure by the restrictor available to the scope while making sure any such modifications are unavailable to the continuation of the discourse.

Example 4 (Dynamic Universal Generalized Quantifier). As for A, we apply EVERY to DONKEY, giving

$$\begin{aligned}
&\vdash \text{EVERY DONKEY} : \delta \to \pi \\
&= \lambda_E.\text{FORALL}\,\lambda_n.\text{DONKEY}\,n \Rightarrow E\,n \\
&= \lambda_E.\text{FORALL}\,\lambda_n.(\lambda_{sk}(\mathsf{donkey}\,(\mathbf{a}\,s)\,n) \wedge k\,(\mathbf{upd}\,s\,(\mathsf{donkey}\,(\mathbf{a}\,s)\,n))) \Rightarrow E\,n \\
&= \lambda_E.\text{NOT}\,(\text{EXISTS}\,\lambda_n.\text{NOT}\,(\lambda_{sk}(\mathsf{donkey}\,(\mathbf{a}\,s)\,n) \\
&\quad \wedge k\,(\mathbf{upd}\,s\,(\mathsf{donkey}\,(\mathbf{a}\,s)\,n))) \Rightarrow E\,n)
\end{aligned}$$

Note that, based on the definition of $\Rightarrow$ in (23), the proposition that the entity $(\mathbf{a}\,s)\,n$ has the property donkey is available in the common ground passed to the consequent $E\,n$.

4 Modeling Definiteness

With our hyperintensional dynamic semantic theory in place, we are ready to extend it to handle definiteness presuppositions in English. We examine both definite pronominal anaphora with *it* and the definite determiner *the*.

4.1 Definite Anaphora with *it*

Rather than adopting an analog of de Groote's `sel` to model English *it*, which cannot possibly select the "right" DR from a left context (see Sect. 1, above), we define dynamic IT : $\delta \to \pi$ as follows:

$$\vdash \text{IT} = \lambda_{Ds}.D\,(\mathbf{def}\,s\,\text{NONHUMAN})\,s \tag{28}$$

where $\mathbf{def}_n : \sigma_n \to \delta \to \omega_n$ is a definiteness operator:

$$\vdash \mathbf{def}_n = \lambda_{sD}.\bigsqcup_{(\mathbf{r}\,s)} \lambda_{i:\omega_n}.(\mathbf{c}\,s)\ \mathsf{entails}\ (\mathbf{cont}\,s\,(D\,i)) \tag{29}$$

(Here $\mathsf{entails}$ is Pollard's (2008a) entailment relation between propositions.) For each $r : \rho_n$, the operator $\bigsqcup_r : (\omega_n \to \mathrm{t}) \to \omega_n$ takes a subset of the first n DRs and returns the unique greatest element (if any) with respect to the (restriction of the) preorder r on ω_n. Thus for a structure s and a dynamic property D, the **def** operator returns the highest DR (if any) in the resolution $(\mathbf{r}\, s)$ whose image under the current assignment $(\mathbf{a}\, s)$ can be inferred from the common ground $(\mathbf{c}\, s)$ to have the staticized counterpart of the property D.

As its name suggests, $\textsc{nonhuman} : \delta$ is the dynamic property of being nonhuman, which is built on the static property $\mathsf{nonhuman}$ similarly to $\textsc{donkey}$ and $\textsc{bray}$ in (18) and (19):

$$\vdash \textsc{nonhuman} = \lambda_{nsk}.(\mathsf{nonhuman}\ (\mathbf{a}\ s)\ n) \wedge k\ (\mathbf{upd}\ s\ (\mathsf{nonhuman}\ (\mathbf{a}\ s)\ n)) \quad (30)$$

As defined in (28), $\textsc{it}$ selects the most salient inferably $\textsc{nonhuman}$ discourse referent from a given structure. We can assume that the static proposition ($\mathsf{every\ donkey\ nonhuman}$) is in every common ground we would consider, where every is as given in Pollard (2008a). This ensures that **def** will allow $\textsc{it}$ to select a donkey as its antecedent, as desired.

To demonstrate, we apply the $\textsc{it}$ to the single-utterance discourse (I), a variant of (D):

(I) Every donkey denies it brays.

With $\textsc{every}$, $\textsc{donkey}$, and $\textsc{bray}$ as given in (27), (18), and (19), respectively, we define $\textsc{deny} : \pi \to \delta$ as the dynamic meaning of the English verb *deny*:

$$\vdash \textsc{deny} = \lambda_{Ansk}.(\mathsf{deny}\ ((\mathbf{a}\, s)\, n)\ (\mathbf{cont}\, s\, A)) \wedge k\ (\mathbf{upd}\, s\ (\mathsf{deny}\ ((\mathbf{a}\, s)\, n)\ (\mathbf{cont}\, s\, A))) \quad (31)$$

which takes a dynamic proposition and returns a dynamic property. Taking $\textsc{every donkey}$ as given in Example 4, the meaning of (I) is then

$$\begin{aligned} &\vdash (\textsc{every donkey})\ (\textsc{deny}\ (\textsc{it bray})) : \pi \quad (32)\\ &= \textsc{forall}\ \lambda_n.\textsc{donkey}\ n \Rightarrow (\textsc{deny}\ (\textsc{it bray}))\ n\\ &= \textsc{forall}\ \lambda_n.\textsc{not}\ (\textsc{donkey}\ n\ \textsc{and}\ (\textsc{not}\ (\textsc{deny}\ (\textsc{it bray}))\ n))\\ &= \textsc{not}\ (\textsc{exists}\ \lambda_n.\textsc{not}\ (\textsc{not}\ (\textsc{donkey}\ n\ \textsc{and}\ (\textsc{not}\ (\textsc{deny}\ (\textsc{it bray}))\ n)))) \end{aligned}$$

To see how $\textsc{it}$ retrieves its antecedent from context, we examine the rightmost application:

$$\begin{aligned} &\vdash \textsc{it bray} : \pi\\ &= \lambda_s.\textsc{bray}\ (\mathbf{def}\ s\ \textsc{nonhuman})\ s\\ &= \lambda_s.\lambda_{nsk}(\mathsf{bray}\ (\mathbf{a}\ s)\ n) \wedge k\ (\mathbf{upd}\ s\ (\mathsf{bray}\ (\mathbf{a}\ s)\ n))\ (\mathbf{def}\ s\ \textsc{nonhuman})\ s\\ &= \lambda_{sk}(\mathsf{bray}\ (\mathbf{a}\ s)\ (\mathbf{def}\ s\ \textsc{nonhuman})) \wedge k\ (\mathbf{upd}\ s\ (\mathsf{bray}\ (\mathbf{a}\ s)\ (\mathbf{def}\ s\ \textsc{nonhuman}))) \end{aligned}$$

Here, $\textsc{it}$ ensures that the argument to $\textsc{bray}$ is the most salient nonhuman entity in the discourse context. Recall from Example 4 that $\textsc{every donkey}$ in (32) updates the common ground passed to $\textsc{deny}\ (\textsc{it bray})$ with the proposition that $(\mathbf{a}\, s)\, n$ is a donkey. Since the meaning of FORALL is based on EXISTS, we have the full reduction of the dynamic meaning of (I) in (32):

$$\begin{aligned}
&\vdash \lambda_{sk}.(\neg\,(\exists\,\lambda_x(\neg\,(\neg\,(\mathsf{donkey}\;x) \wedge (\neg\,(\mathsf{deny}\;x\;(\mathsf{bray}\;(\mathbf{a}\,\varsigma)\;(\mathbf{def}\;\varsigma\;\textsc{nonhuman}))\\
&\quad \wedge \mathsf{true}) \wedge \mathsf{true}) \wedge \mathsf{true}) \wedge \mathsf{true})) \wedge \mathsf{true}) \wedge k\; s\\
&= \lambda_{sk}.(\neg\,(\exists\,\lambda_x(\neg\,(\neg\,(\mathsf{donkey}\;x) \wedge (\neg\,(\mathsf{deny}\;x\;(\mathsf{bray}\;x) \wedge \mathsf{true})\\
&\quad \wedge \mathsf{true}) \wedge \mathsf{true}) \wedge \mathsf{true})) \wedge \mathsf{true}) \wedge k\; s\\
&\equiv \lambda_{sk}.\neg\,\exists\,\lambda_x.\neg\,(\neg\,(\mathsf{donkey}\;x) \wedge (\neg\,(\mathsf{deny}\;x\;(\mathsf{bray}\;x)))) \wedge k\; s
\end{aligned}$$

where $\varsigma = (\mathbf{upd}\,(\mathbf{intro}\,s x)\,(\mathsf{donkey} x))$ is the structure passed to DENY (IT BRAY).

Thus IT selects its antecedent based on its definiteness presuppositions, yielding the desired truth conditions for (I). The definition of IT in (28) also models the infelicity of (C), where *it* is used without a salient antecedent, because there is no DR that can be inferred from context to be nonhuman.

4.2 The Definite Determiner

We also use **def** to model the English definite determiner *the* as THE : $\delta \to \delta \to \pi$:

$$\vdash \textsc{the} = \lambda_{DEsk}.(\lambda_n(D\,n\;\textsc{and}\;E\,n)\;(\mathbf{def}\;s\;D))\;s\;k \tag{33}$$

This translation of *the* resembles the indefinite determiner A in (25) in that the meanings of the dynamic properties D and E are composed via AND. The main difference is that while A uses EXISTS to introduce a new DR, THE uses **def** from (29) to select the most salient DR from the discourse context with the property D. Using AND to pass this DR to both properties ensures that any modifications to the structure that result from D are inherited by E.

In the discourse in (J), a simplification of (A), (B), and (F), the noun phrase *the donkey* can only refer to one of the discourse referents introduced prior to its use:

(J)
1. A donkey enters.
2. A mule enters.
3. The donkey brays.

To model this discourse, we first define the dynamic properties MULE : δ and ENTER : δ similarly to the definitions of DONKEY and BRAY in (18) and (19):

$$\vdash \textsc{mule} = \lambda_{nsk}.(\mathsf{mule}\;(\mathbf{a}\;s)\;n) \wedge k\;(\mathbf{upd}\;s\;(\mathsf{mule}\;(\mathbf{a}\;s)\;n)) \tag{34}$$

$$\vdash \textsc{enter} = \lambda_{nsk}.(\mathsf{enter}\;(\mathbf{a}\;s)\;n) \wedge k\;(\mathbf{upd}\;s\;(\mathsf{enter}\;(\mathbf{a}\;s)\;n)) \tag{35}$$

With A and THE as in (25) and (33), we have the following dynamic meaning for the discourse in (J):

$$\vdash ((\textsc{a donkey})\;\textsc{enter}\;\textsc{and}\;(\textsc{a mule})\;\textsc{enter})\;\textsc{and}\;(\textsc{the donkey})\;\textsc{bray} : \pi \tag{36}$$

We start with the leftmost conjunct of the discourse:

$$\begin{aligned} &\vdash (\textsc{a donkey})\ \textsc{enter} : \pi \\ &= \lambda_{sk}.\exists\, \lambda_x.\mathsf{donkey}\ x \wedge \mathsf{enter}\ x \wedge k\ (\mathbf{upd}\ (\mathbf{upd}\ (\mathbf{intro}\ s\ x)\ (\mathsf{donkey}\ x))\ (\mathsf{enter}\ x)) \end{aligned} \tag{37}$$

Combining the term in (37) with the entire left conjunct of the discourse, we have:

$$\begin{aligned} &\vdash (\textsc{a donkey})\ \textsc{enter}\ \ \textsc{and}\ \ (\textsc{a mule})\ \textsc{enter} : \pi \\ &= \lambda_{sk}.\exists\, \lambda_x.\mathsf{donkey}\ x \wedge \mathsf{enter}\ x \wedge (\exists\, \lambda_y.\mathsf{mule}\ y \wedge \mathsf{enter}\ y \wedge k\ \varsigma) \end{aligned} \tag{38}$$

where ς represents the structure that results from the application of AND in (38):

$$\varsigma = (\mathbf{upd}\ (\mathbf{upd}\ (\mathbf{intro}\ (\mathbf{upd}\ (\mathbf{upd}\ (\mathbf{intro}\ s\ x)\ \mathsf{donkey}\ x)\ \mathsf{enter}\ x)\ y)\ \mathsf{mule}\ y)\ \mathsf{enter}\ y) \tag{39}$$

The right conjunct then uses THE to select the most salient DONKEY from the preceding discourse, applying BRAY to it:

$$\begin{aligned} &\vdash (\textsc{the donkey})\ \textsc{bray} : \pi \\ &= \lambda_{sk}.(\lambda_n(\textsc{donkey}\ n\ \ \textsc{and}\ \ \textsc{bray}\ n)\ (\mathbf{def}\ s\ \textsc{donkey}))\ s\ k \\ &= \lambda_{sk}.(\textsc{donkey}\ (\mathbf{def}\ s\ \textsc{donkey})\ \ \textsc{and}\ \ \textsc{bray}\ (\mathbf{def}\ s\ \textsc{donkey}))\ s\ k \\ &= \lambda_{sk}.\mathsf{donkey}\ (\mathbf{a}\ s)\ (\mathbf{def}\ s\ \textsc{donkey}) \wedge \mathsf{bray}\ (\mathbf{a}\ s)\ (\mathbf{def}\ s\ \textsc{donkey}) \\ &\quad \wedge k\ (\mathbf{upd}\ (\mathbf{upd}\ s\ \mathsf{donkey}\ (\mathbf{a}\ s)\ (\mathbf{def}\ s\ \textsc{donkey}))\ \mathsf{bray}\ (\mathbf{a}\ s)\ (\mathbf{def}\ s\ \textsc{donkey})) \end{aligned} \tag{40}$$

Since the structure ς passed to (THE DONKEY) BRAY by the preceding discourse is such that $((\mathbf{a}\ \varsigma)\ (\mathbf{def}\ \varsigma\ \textsc{donkey})) = x$, we arrive at a final reduction of the term in (36) that models the entirety of the discourse (J):

$$\lambda_{sk}.\exists\lambda_x.\mathsf{donkey}\,x \wedge \mathsf{enter}\,x \wedge (\exists\lambda_y.\mathsf{mule}\,y \wedge \mathsf{enter}\,y \wedge \mathsf{donkey}\,x \wedge \mathsf{bray}\,x \wedge k\,\varsigma') \tag{41}$$

where $\varsigma' = (\mathbf{upd}\ (\mathbf{upd}\ \varsigma\ (\mathsf{donkey}\ x))\ (\mathsf{bray}\ x))$ is the structure extending ς that results from applying AND to the left and right conjuncts of the discourse ((38) and (40)).

Note that, in this example, the dynamic definite determiner THE picks out the most salient DR from the structure it is given that has the property specified as its argument (i.e., DONKEY). However, as (F3) shows, substituting the pronoun *it* for *the donkey* makes the discourse infelicitous. Our theory captures this infelicity because IT only requires its antecedent to have the property nonhuman, which is weaker than the property donkey with respect to entailment. With IT replacing THE DONKEY in the right conjunct of (36), **def** would be incapable of selecting a unique nonhuman DR from ς since two DRs would then have the property NONHUMAN (namely, the mule and the donkey). The infelicitous examples in (E) through (H) can be ruled out for similar reasons.

5 Conclusion and Future Work

We have presented a dynamic theory of discourse meaning formulated in higher order logic that incorporates aspects of de Groote's continuation-based theory and Pollard's hyperintensional semantics. Drawing on the work of Heim and Roberts, our theory provides an enriched notion of discourse context that includes discourse referents ordered by relative salience and a common ground of mutually accepted content. We have shown how this enriched context allows the definiteness presuppositions in English associated with the pronoun *it* and the determiner *the* to be captured in a way that is faithful to the facts. The resulting theory repairs the inadequate treatment of anaphora resolution in de Groote's work based on the oracular `sel` function.

In future work, we will continue our collaboration with Roberts and Smith on developing a general, categorially based theory of projective meaning. The next avenues for this research include spelling out how relative salience is adjusted by re-invoking a previously introduced DR (see the discussion of (A) and (B) in Sect. 1) and integrating the hyperintensional dynamic semantic theory introduced here with a fully compositional theory of English grammar that takes e.g. quantifier scope, unbounded dependencies, and intonation into account.

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